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L'objectif de ces sessions d'étude était de proposer des recommandations en ce qui a trait à la formation en mathématiques de trois catégories de décrocheurs précoces et également de dégager des orientations pour la recherche future. Les questions suivantes ont été abordées: comment dépister les élèves qui risquent d'abandonner prématurément? quelles stratégies permettent d'atténuer ce risque? comment rejoindre les élèves qui ont déjà décroché? quel contenu mathématique doit-on leur proposer? quels niveaux de rendement peut-on attendre d'eux? quelles sont les stratégies d'enseignement les plus adéquates? et quel est le matériel disponible?

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PROCEEDINGS OF THE WORKING GROUP (WG 19)

ON MATHEMATICS FOR PREMATURE SCHOOL-LEAVERS

7TH INTERNATIONAL CONGRESS
ON MATHEMATICAL EDUCATION

(ICME VII)

QUÉBEC, 1992

CIRADE - UQAM

EDITORS:
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RICHARD PALLASCIO

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MATHEMATICS FOR PREMATURE SCHOOL LEAVERS

Carlos Vasco
Chief organizer, Colombia

Introduction

The goal of the four sessions was to propose a set of recommendations related to mathematics education for three populations of premature school leavers, and to propose problems and guidelines for further research. The three populations may be labeled "very early school leavers" (VESL), "early school leavers" (ESL), and "late, but still premature, school leavers" (LPSL), depending on whether they leave school before, during, or towards the end of high school. The Group addressed the following questions:

(1) How can students at risk of leaving early be detected?

(2) What strategies minimize this risk?

(3) How can students who have already dropped out be reached?

(4) What mathematical content should be considered?

(5) What achievement levels should be aimed at?

(6) What are appropriate teaching strategies?

(7) What materials are available?

MATHEMATICAL EDUCATION OF ILLITERATE ADULTS

Interaction of outside school constructed mathematical knowledge with school mathematization

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Introduction

One great problem of a nation like Brazil nowadays is to solve the question of its 20 million illiterate young people and adults who have the right to acquire basic systematic knowledge transmitted by the school. Many Brazilians in rural zones leave school or don't have access to it when they are children (Freitag - 1991). A large percentage of these people have migrated to the big cities in search of better jobs, better living conditions, or even just getting to know different places. São Paulo is a typical case of a city where a large part of the population is not native (IBGE - 1991). These adolescents and adults, faced with the problem of surviving in a complex industrialized society, feel the need of schooling. They look for supplementary courses or other courses concerned with the education of young people and adults.

In one way or another, these young people and adults are adapted to society in spite of being illiterate, not able to codify or decodify written languages. However, in order to carry out their everyday activities, in the professional or personal spheres, these people are forced to mathematize, that is, to establish relationships of a quantitative and/or geometric nature. However, traditionally, the school has refused to accept the mathematization carried on outside the classroom; either because they don't accept it as mathematical knowledge (Gerdes - 1984) or because they are unable to produce teachers capable of integrating it with systematized mathematics (Carraher - 1989).

The studies concerned with the mathematics constructed outside the classroom have shown how people make use of sometimes very complex and even sophisticated mathematical procedures (1) in the resolution of problems which appear in everyday situations. My research program aims at operationalizing methodological principles which

will enable the student to integrate everyday mathematical procedures with school mathematical knowledge. Basically, I am trying to face the classroom's complexity, taking into consideration the social relations involved. In this task, I selected for consideration not only the social relations generated in the class-society in which we live, but also the ones the school system itself causes, as Souza (1953) underscores for us. These are the role of instructor, reader, speaker, etc. and also the roles of bad and good students in the school population (2).

If we find the denial of mathematical knowledge acquired outside school in regular courses, the situation becomes even more serious for those concerned with the education of young people and adults. The causes of this aggravation are several: specially the great diversity of the students' experiences and the precarious training in mathematical education of the primary school teachers.

In this picture mathematics, while a school subject-matter, has contributed to the selectivity of Brazilian schools because it has a close relation with the conception of mathematical knowledge which permeates the teacher's pedagogical practice (Carvalho - 1989). This science is considered to be a finished and perfect area of knowledge, pertaining only to the world of ideas. Thus, the structure of mathematic's systematization must serve as the model for the other sciences.

The consequence of this conception, perhaps the most disastrous result, is that school success in mathematics is used as a criterion for the appraisal of the students' intelligence. Few pupils, only the more intelligent, succeed in the comprehension of such an elaborate science; the others must train themselves in procedures which they don't understand. The students who don't succeed in this training will be excluded from the school (Martins - 1984). As many educators, who work with different perspectives and several lines of research (Mello - 1984; Carvalho & Onaga - 1985; Carraher - 1989) emphasize, once more, the victims are transformed into culprits by their exclusion from the school system.

In the courses aimed at adult education, to the traditional conception of mathematical knowledge, are added the prejudices with regard to poor students and those who were, at some time, excluded from the school system or were not included at the regular age. The pupils are catalogued as incompetent before the first school day; the teacher assumes his/her task as a philanthropic act and not as a social and political function which he/she must assume professionally.

Even the teacher who faces up to the political aspect of supplementary work, does this as a participant in politics and not as a professional teacher. The honorable exceptions have found little help either financial or administrative (3) for the development of their work.

Moreover, there is the problem of the so-called "cultural respect". When the school refuses to accept the mathematical knowledge acquired by the child outside the classroom, only a part of his/her life is denied. But if that situation happens with an adult coming to supplementary courses, it is his/her whole life experience and his/her culture that are put under suspicion.

If we want to build up a method of teaching mathematics which makes it possible to surmount the previously mentioned problems we will have to reformulate our conception of knowledge, taking on another, more dynamic, more dialectic one (4). However, we can't deny, as schools have been doing, the condition of intellectual to all human being, the capacity to acquire and construct culture in the process of social interaction. As Gramsci (1986) points out, the man/woman who doesn't perform the professional role of an intellectual and acts practically, does not cease to have a knowledge of the world, moreover he/she transforms it.

From birth, every human being begins his/her process of quantification of the world and exploration of space, constructing, in his/her interactions with reality his/her knowledge of the common sense, in Gramsci (1989) terminology. The competence of the teacher is to make it possible for the students to incorporate their earlier experiences concerning the concepts studied at school and their interpretation of them.

It is worth pointing out that I am not assuming a "spontaneist" position. I think that the human being constructs instruments in the process of mathematization of the situations he/she encounters in everyday life. Those instruments are contextualized, that is, restricted to that situation. Furthermore, often the individual is not conscious of them. It is the specific role of the school to intervene in order to make the students aware of the instruments at their disposal and to discontextualize them, making them more abstract, general and complex. Such a process of learning should make the students, having these instruments, capable of returning to reality, of explaining it, reorganizing it and transforming it. In other words they can mathematize it again but in an increasingly more scientific way.

This position rejects any type of differentiation between the "science" to which the student from the less-favored classes of the population has access and that of those students whose families have greater social and economic resources. Both groups of students have to be in contact with what is understood as the core of each science. What must be transformed is the situation in the classroom, in order to incorporate a more dynamic conception of mathematical knowledge.

As the great Brazilian educator Paulo Freire (1986) points out, teachers cannot be elitists and there are two ways of being this. One is to impose their language as the only valid one. The other form of elitization of knowledge is to bring one's speech down to the student's level, impoverishing it, making a caricature of it. We must succeed in being simple without being simplistic, because simplistic language makes caricatures of the students as stupid, as unable to understand us. Furthermore it devaluates the subject studied and, by devaluating it, we are devaluating the students and this is a form of elitism.

Methodological principles

Franchi and I have systematized the methodological principles which orient mathematics teaching, according to the perspective expressed before (Franchi & Carvalho - 1984/1992). In that article, we illustrate the above principles with activities carried out in the classrooms of regular courses. However, it is necessary to restructure those

methodological principles making them operative in this other classroom situation. One principle refers to the right of all students to acquire mathematical knowledge useful as a universal knowledge (Franchi & Carvalho - 1984/1992).

The incorporation of mathematization carried on outside school with the knowledge constructed in the classroom runs into difficulties of different types. One concerns the acceptance of this principle by the teacher. Even when there is acceptance, the teacher has difficulty in identifying the mathematical properties in the student's procedures and in representing them in mathematical language. Another difficulty concerns the negotiation which the teacher has to carry on with the students in order to have them accept as knowledge the procedures which they use in their daily activities because the image which the adult illiterate students have of school, even if they have never attended one, is a traditional one (where a wise teacher speaks and an ignorant student listens and absorbs).

Another type of difficulty comes from the students' irregular degree of schooling and the diversity or their life histories. In the same classroom, students with different levels in the system of production and, therefore, with different degrees of mathematization interact with each other. Besides this, in classes dedicated to adult education in an urban centre in a country like Brazil, the scholastic experiences through which the students have already passed embrace a wide gamut: their mathematical knowledge, therefore, has various degrees of conceptualization and systematization.

One could suppose that the solution for adult education would be the preparation of instructional modules which would overcome the difficulties pointed out. However, the construction of mathematical knowledge is a long and slow process, and, it is an eminently social activity (Franchi & Carvalho - 1984/1992). The graphic representation of their procedures by the students requires them first to verbalize them. As pointed out by Vygotski (1977, 1979, 1984 and 1988) and Wallon (1986 and 1989), it is necessary to have not only a consistent theoretical base, but also to elaborate activities which promote teacher/student and student/student interactions mediated by speech.

Some educators have questioned the use of the so-called "concrete materials" in mathematical learning activities (Fiorentini & Miorim - 1990). We have already referred to the necessity of putting the emphasis of the activities on the operation carried out and not on the objects (Franchi & Carvalho - 1984/1992). This principle remains valid in adults' education. However, such objects, as we have also already said, can be of different levels of representation -- letters, numbers, drawings, copies of paper money, diagrams (Franchi & Carvalho - 1984/1992). It is in this aspect that the consideration of the work of students with irregular schooling takes its own path. First, in their daily lives, the situations involving mathematics, for example operations with money, especially in urban centres, are very "concrete" although the value of coins and bank-notes is symbolic. Besides that, the solution of the problems arising from activities which use structural material (Montessori's Golden Material, Dienes's Logic Blocks, abacuses, etc.) require a knowledge of the rules of the material's structure, which are of a symbolic nature. The illiterate adult is not always familiar with games which involve this type of action.

It might appear that I am defending not using any material which is not familiar to the students. However, often, the students daily life doesn't offer a sufficiently rich source of material to broach some aspect of the subject matter which one wants to work with. Then the teacher, as a member of the group, carrying out his/her specific role, takes a cultural object from his reserve into the classroom.

A principle implicit in this last paragraph is the need to broach the same content under all the mathematical aspects concerned with it.

A didactic of mathematics based on these principles cannot help but suggest an undertaking where, with the passage of time, a subject is approached at different levels of complexity, with different types of representation, including those used by mass media: tables, graphics, etc. (Franchi & Carvalho - 1984/1992).

Under these circumstances, rigor becomes an effect of the activity arising from the need to communicate and is not a previous condition (Franchi & Carvalho - 1984/1992).

Content organization and programming of activities have to take on a much more flexible character, because we must consider that as the construction of Mathematics was not a linear process, neither is the acquisition of mathematical knowledge (Franchi & Carvalho - 1984/1992). We do not think that the learning process is the same, following the same stages, or is even parallel with the historical process. However, the historical process gives us important guidance on the non-linearity of knowledge construction and on the aspects which must be taken into account in the study of this concept (Caraça - 1984). The needs of the school community may often superimpose themselves on the order of the contents and/or the activities defined a priori by the teacher.

Once more the educational act is characterized as a continual process of action-thought-action (Franchi & Carvalho - 1984/1992). From this point of view, the adult student shows a greater resistance to participating in intellectual work, perhaps because, in many cases, it is the first time that he/she has been invited to participate in this type of activity. In compensation, perhaps just because of this, he/she shows more pleasure in being able to accomplish his/her tasks successfully.

School mistakes lose their connotation of a failure to be hidden when they are considered as raw material, useful in the process of knowledge construction in which teacher and students are engaged together (Franchi & Carvalho - 1984/1992). This is another point for negotiation between the teacher and his/her adult student. In Brazil, the students did not make this choice of having an irregular schooling. It was the circumstances of their lives, generally the necessity of working, which brought them into this situation. That student already has a life punctured with failures and it is very difficult for him/her to record a mistake which will mark him/her as material for recycling.

The principles propounded lose their significance if a new process of evaluation is not adopted by the teacher. The evaluation of the learning process ceases to be capable of being performed at any single moment at which the teacher judges the acquisition of mathematical knowledge by his/her students by the final product. It should be an integration of all the productive moments (Franchi & Carvalho - 1984/1992). In each class:

— The student evaluates himself/herself checking his/her results with other similar situations or in other mathematical procedures.

— The group evaluates itself and evaluates each student by discussing the different procedures which came up in class, choosing the most efficient in that situation, the most general, the simplest...

— The teacher evaluates the adequacy for that class of the activities, the topic, the content chosen, etc.

I am aware that this proposal comes up against conflicts generated by the contradictions which emerge in the Brazilian school system. The overcoming of these contradictions causes the appearance of others which must cause further changes and therefore this transformed school will never be ready and finished. However, it is in the perspective of its construction that, from my point of view, the work of all educators should be directed, especially if it is directed to the schooling of the underprivileged population.

Notes

(1) Many scientists are dedicated to studying the mathematical knowledge constructed outside the classroom. In Brazil, a group of scientists mention the existence of oral mathematics, not standardized (Carraher - 1986, 1988; Lima - 1986; Schliemann, Santo & Costa - in press). Borba (1987) had the opportunity to research the mathematical knowledge implicit in the activities of the community of the favela at Vila Nogueira, in Campinas, aiming at elaborating a pedagogical proposal from an informal school. Carvalho (1991) studied the mathematical knowledge elaborated by the Rikbaktsa, an indigenous ethnic group which lives in the Central-West region of Brazil. There are also scientists who study the cognitive development of Brazilian adults with low incomes (Oliveira - 1983, 1986) and illiterate (Tfouni - 1988). There are studies which specifically approach the courses concerned with Adult Education (Haddad - 1982, Oliveira -1987 e Oliveira & Duarte, 1990) and the mathematical education in these courses (Duarte - 1987; Oliviera, Hara & Meirelles - s/d; Souza - 1988).

(2) The social-interactionist research tradition is Russian (Vigostsky - 1977, 1979, 1984, 1988; Leontiev - 1977, 1988; Luria - 1977, 1988, 1990), however they are not the only ones to follow this perspective of study (Mead - 1922; Wallon - 1986, 1989). They conceive people as basically social beings, introduced into an historically constructed culture. They point out the social construction of knowledge.

(3) Until very recently, the official school system explicitly assumed this philanthropic characteristic of the teacher's function: the municipal supplementary courses of the city of São Paulo were under the responsibility of the Secretary of Social Welfare until 1989. Only at this time did the Municipal Secretary of Education assume them.

(4) To assume this position professionally implies taking a conception of mathematics as a science in constant construction, whose postulates and laws are always provisional. I agree with Caraça (1984) when he states that mathematics presents us with a braid of truth and error, an area of knowledge where brilliant triumphs and great failures live side by side. All knowledge's constructors are social beings; they are born, live and develop physically and mentally in a group of human beings which has its own ideology and culture. Therefore, the meaning of the object of knowledge, whatever it is, is determined by the historico-cultural conditions of the society in which this object is inserted and with which it is in constant interaction.

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INSTRUMENTAL MATHEMATICS AND CONTEXTUALIZATION

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Introduction

Whether it be in developing or industrialized countries, mathematics education often fails to achieve its objectives. Pupils don't grasp the relevance of its teaching, don't see its usefulness, lose interest in this knowledge, and try to avoid job areas where a good mathematical training is required ... or leave school. In some countries, if it is not possible for us to keep pupils in mathematics after elementary school, at least when they are there, we can try to get them to see mathematics as a tool for solving many kinds of problems. We propose two ideas, the first involving learning based on contextualized activities, the other involving an instrumental conception of mathematics, which results in significant and meaningful learning for students.

Contextualization and learning

Contextualized reasoning functions in opposition to the abstract or general thinking that schools try to develop in students. While the educational system tries to supply generalizable knowledge that students can apply at some future date in wide and varied contexts, some research has shown that this aim of transmitting general knowledge is often contradicted in practice.

Contextualized reasoning corresponds to a more cognitive approach to knowledge and leads naturally to the role of internal and external representations in the thinking process. Janvier (1990, 1987) shows, for example, that in his contextualized thinking the

of the mathematical relationships involved. In other words, to understand it is necessary to have first hand experience, and to justify or think about a result, it is necessary to have understood. Experience precedes understanding, which itself precedes judgment, the latter leading possibly to responsible action.

Thus the operations schema of intentional consciousness, according to Angers and Bouchard, consists of four operational levels: experience, intelligence, reasoning and responsibility, as well as the functional relations between them (figure 1). «The sensory perceptions - vision, touch, audition, taste - gather some facts and artifacts, but they don't reveal intelligible connections between facts and artifacts, and they don't reveal the consequences of this understanding. Reflection turns back on the previous operations and complete them by a critical and detached elaboration by checking to distinguish the true from the false, correct or inaccurate, certain or probable in the understanding. Finally, after a judgment on the facts, deliberation occurs to decide what we can do with respect to gathered data, acquired understanding and produced judgment.» (id., 1986a, p. 49-50)

The mathematical tool

Teachers are beginning to appreciate the importance of developing this rationality, as judged by the proposals made at a General Meeting of mathematics teachers in Québec: "the pupil must learn to use mathematical reasoning by formulating hypotheses, selecting proof elements, constructing arguments, extracting conclusions, mastering the mathematical pathway" (Pallascio (dir.), 1990).

Now, we all have a personal conception of mathematics. Bernard Charlot (1976) summarizes these with three different epistemological conceptions: the "heavenly mathematics" (an expression proposed by the philosopher of science Jean T. Desanti), i.e., perceived as pre-existing structures, the "earthly mathematics", perceived as the structure of the natural or social world, and mathematics perceived as instruments - as a creation.

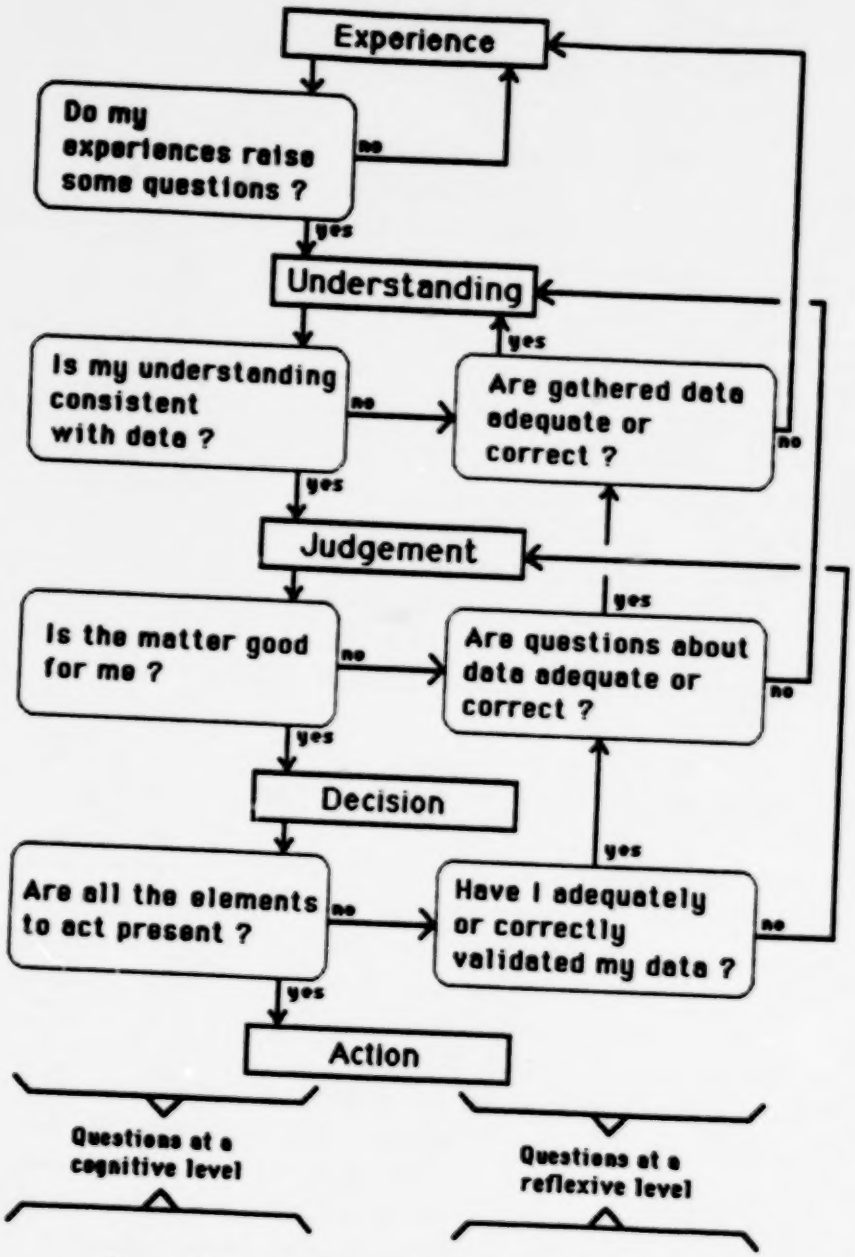

Figure 1: Functional relations between the learning levels, according to Angers and Bouchard's schema

The pedagogue holding the first conception, the most widespread view, seeks to present the mathematics world as existing independently from our thought, which must be fashioned like them (i.e., "take the form of"). According to the second conception, mathematics exists in the things themselves. By concrete manipulation, the child will eventually appropriate the mathematical elements they contain. It is only required to put in sufficient time. According to the last conception, mathematics has not been discovered, but has been created, invented. It is a metaphor of the real world among others, for example, the artistic world (Pallascio, 1991).

Projects with contextualized activities

Therefore we advocate that the learning of mathematics be built on contextualized activities which allow students to experiment directly with real phenomena and to solve interesting problems with the help of the tools of mathematics: data processing, descriptive diagrams, algorithms on numbers, successive approximations, trial and error ... In primary school a project based pedagogy seems best suited for this.

The carrying out of projects, (to "project" oneself), is to become free of the present moment, and appeal to one's own experience. By means of a project, a person can emerge and pass beyond the present moment by his or her own thought, and this allows one "actually" to see into the future. By this vision, our intelligence enables us to foresee, in a sense. A project is simultaneously a projection and an answer to ideals which we can feel internally. Life is a global project, inseparable from the small or large projects that compose it (Castonguay and Pallascio, 1978).

Human beings act, set actions where initiatives can be taken and where there is mastery of their action. In fact, we act when we make a decision and when something is desired that pleases us. To be sure a person encounters his or her capacities and inadequacies when facing their project. Even the human body is submitted to spatial and temporal constraints. But two factors allow us to move ahead all the same. First, we all go through the influence of other human beings, by which we are inspired to go forward. Second, learning allows us to escape from many constraints. The dynamics of our will can change inadequacies and capacities into new capacities, and can evolve toward our

realization. Thus, the realization of projects becomes a human activity we live everyday. To live, is to project! (Pallascio, 1992).

Mutual common aid favors the acquisition of knowledge. The culture of a people and the values that support it, favor a life apprenticeship within the community. This becomes educative. For example, some of life's rules emerge from conflict situations. But they will have the advantage of having been felt through the experience of the young.

Children can educate and instruct themselves according to the same rules and at the same rhythm as adults: reading, experimenting, building, inventing, creating ..., and by so doing apprehend science and culture according to their own possibilities and their own needs. Children can thus structure their own knowledge. They can carry out tasks that place them in real life situations that give them the habit of serious and productive effort. Thus the children can learn to build their own nest, even if they don't know how to build it perfectly on the first try. Progress will never eliminate trial and error in the course of constructing their personality. From this point of view, children are emerged in life and educate themselves by living (Pallascio, 1986).

We will briefly illustrate the different phases of the pupils' course during their integrative projects (see Angers and Bouchard, 1984). These phases interact and overlap. The first four phases (see the circle in the center of the figure 1) are effected normally only one time at the beginning of the year, while the four last phases (see the dotted ring on the figure 2) can be carried out a number of times, according to the childrens' age, the scope of the selected projects and the production rate of the teams.

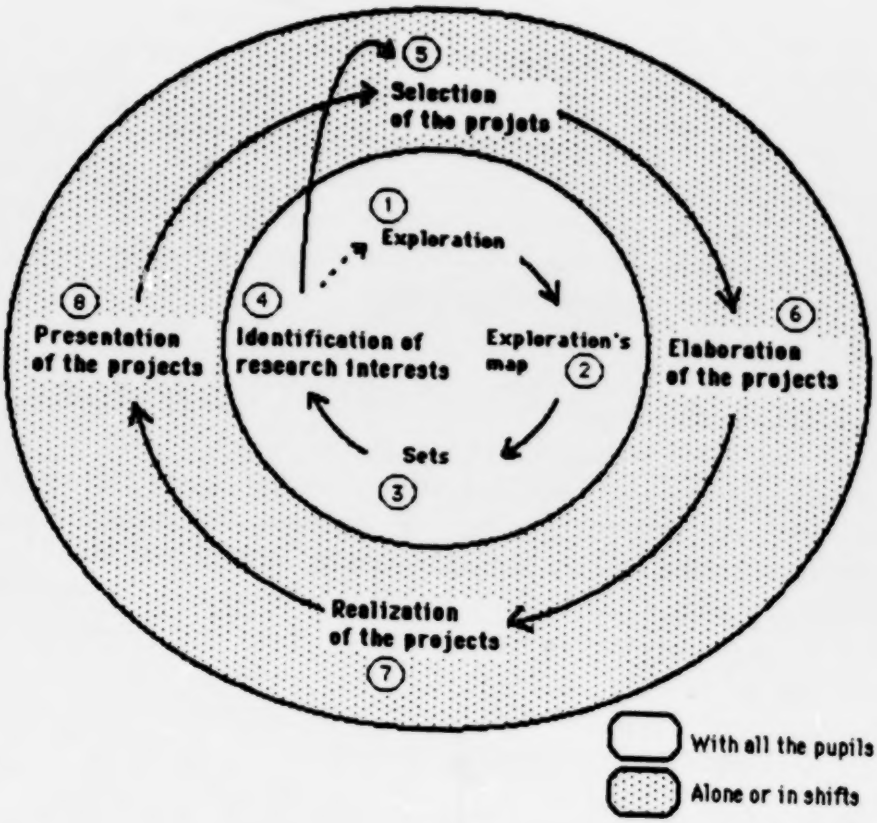

Figure 2: Integrative project's phases

There is no competition based on the number of projects effected in a year; quality being preferred to quantity. The children are the architects of their integrative projects. This pedagogy of projects is formal in regard to the methods of work, but opens according to the contents of the childrens' interest. At each phase the child organizes his or her actions with the help of his or her diary, identifies the goal objectives and estimates his or her level of attainment with the help of guide posts.

An example of contextualized activity

This example concerns a team composed of two ten year old pupils involved in a space project. One pupil had read the report of an experiment conducted in an American space shuttle, that showed that the growth of tomato plants was accelerated in the absence of gravity. The hypothesis the pupil formulated was: in a similar manner, the human brain can function more efficaciously in space because the human being can solve problems more rapidly in space...!

Our interventions were brought to bear on the hypothetical nature of this question and on the way to address the question at the level of the children. We finally arrived with them at the necessity to proceed by analogical reasoning. The experimental design consisted of asking other children to solve a tangram, a famous mathematical puzzle. The voluntary pupils were placed in different contexts: a "cosmic" silence versus a noisy environment obtained by "heavy metal" music. The perspicacious reader will readily note a cognitive slippage from space vacuum to silence, but the analogy is close enough to go ahead with this analogical experiment. The pupils calculated the means of the two experimental groups in regard to the time required to solve the problem. On the basis of the results they were obliged to reject their starting hypothesis.

The recognition of the hypothetical status of their intuition, the determination of the criteria that allowed them to validate the value of their hypothesis in its analogical context, even the refutation of their hypothesis, while at best they could only have "not rejected" it (we don't accept a hypothesis, but don't reject it, thus recognizing the possibility of error). These are some of the elements that allow working knowledge at a rational level, i.e., in the terms of the operational scheme of intentional consciousness, beyond a strictly intellectual knowledge.

Conclusion

First hand experience is irreplaceable. Empirical knowledge is the very foundation for the theory of knowledge applied to project-based pedagogy described above. Too often teachers are content to have students read texts which relate experiments done by others. This is not sufficient.

Of course, at times, as was the case in our example, to verify a hypothesis which seems far-fetched or impossible to verify, we need to resort to a simulated experiment. We proceed by analogy. And the analogies can be more or less good or more or less realistic. Everyone has played games simulating the stock market, auctions, etc. Even if the simulation is not reality, we can easily agree that it does teach us a lot about the subject.

In this type of analogical experiment, it is obvious that the richness of the simulation will be proportional to the number of analogous factors present. Here the children only considered the "silence" factor found in space. Had they more fully explored the environmental characteristics of space travel (weightlessness, physical and mental preparation, scientific research laboratory, training ...), no doubt their experiment would have provided more information for a better understanding and judgment of the particular situation being studied. "Understandable knowledge is essentially usable knowledge" (Aubé, 1974).

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RAISING ACHIEVEMENT IN MATHEMATICS PROJECT (1986-1989)

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Context

The Raising Achievement in Mathematics Project (RAMP) a curriculum development research project was funded jointly by the department of education and Science and 34 local education authorities in England. RAMP operated through a network of five regional centres. At the end of three years, some 8000 teachers were involved nationally.

During the Project's life the national focus an achievement in mathematics has led to a vigorous debate. Reports and surveys, such as the Cockcroft Report, Mathematics Counts (1982) have traced this concern over the past century and have shown that school leavers lack confidence and are unable and unwilling to apply the mathematics they have been taught. The Low Attainers in Mathematics Project 1983-86 (the first phase of RAMP) set up in response to the Cockcroft Report, was charged in the first instance with the task of developing 'good practice' in the teaching of mathematics to low attaining pupils and then of investigating methods of disseminating its findings. One of the main findings of LAMP was that underachievement is a problem not only for pupils in the lowest 40% attainment range, but also concerns the majority of pupils and teachers.

RAMP was multi-dimensional. The central aim was to raise standards of pupil achievement in mathematics, at another level it was involved in developing the mathematics curriculum and at a third level it contributed to the development of an in-service model for teachers which has proved less disruptive for schools.

The processes by which change might be implemented

The essence of the Project's work and findings is that the route to raising pupils' achievement and developing effective classroom practice is through encouraging teachers' confidence in their own professionalism. This process does not involve a sudden introduction of new schemes, syllabuses, resources, organizational structures or an adoption of a particular teaching style. Therefore, our work has been mainly concerned with how teachers can be enabled to fulfill this responsibility effectively and economically.

For the Project's aim of improving mathematics teaching to be realized, any actions taken had to be embedded within the classroom situation. Thus the authority for carrying out research and development lay with the teachers. 70 teacher-researchers were released from their teaching commitment for one day a week over three years and participated in regional and national weekend conferences. This structure enabled the teachers to sustain all the day to day problems of school life, such as difficult pupils and conditions, report writing, examinations and marking. On this day they worked in a number of ways including:

- working together, discussing, sharing and reflecting on classroom experiences both successful and unsuccessful;
- working in groups, focusing on specific issues, including the examination of a number of previous relevant curriculum development and research projects, both in mathematics and in other curriculum areas;
- developing strategies for in-service work in their schools and with other teachers;
- exploring mathematical situations and evaluating possible outcomes;
- evaluating the effectiveness of commercial resources in the classroom;
- exploring the creative use of calculators and microcomputers in the mathematics classroom;
- working with other teachers on in-service courses;
- working with other teachers in their classrooms.

On their four days in school the teacher-researchers explored possibilities and ideas within their own classrooms, involved their colleagues through discussion and collaborative teaching and kept personal records.

To ensure the evolution of attitudinal changes and to minimize the constant need for 'retraining' the RAMP directorate exercised its coordinating role through:

- facilitating continuous dissemination evidenced by the large network of teachers involved;
- encouraging experimentation and critical appraisal of classroom practice;
- assessing the strengths and weaknesses of curriculum initiatives, identifying factors which could limit regional and national developments and suggesting strategies for improvement;
- increasing the pool of people who can act as a regional resource base for in-service support and curriculum development;
- developing and supporting Centres to provide a regional leadership in curriculum development and assessment policies.

The goals and content of the project

The main aim of the Project was to raise achievement in mathematics for all pupils involved.

To this end, the Project aimed to effect sustained development by:

- recognizing that learning is a natural human endeavor whose exploration is essential for both pupil and teacher development;
- encouraging professional growth of teachers through providing opportunities for teachers to reflect, to exchange experiences and to evaluate their current classroom practice;
- providing opportunities for teachers and their schools to become more critical, economic and effective users of resources, in-service provision and curriculum development initiatives;

- enabling pupils to apply their knowledge and skills in mathematics to other school subjects and to life in general;
- identifying and articulating 'good practice' so that external agencies such as parents, governors and employers could appreciate the nature and purposes of the Project and share responsibility for continual improvement in pupils' learning of mathematics.

So RAMP set out:

- to explore the reasons for underachievement;
- to identify and communicate what constitutes raised achievement in mathematics;
- to propose strategies for raising pupils' achievement;
- to investigate methods of disseminating the Project's findings.

The Project collected case studies and supportive quantitative data as a record of pupils' progress in mathematics and of strategies which facilitated this.

The student perspective

Mathematics has enjoyed prestige as a subject for a very long time, however, there is a wide range of definitions as to what mathematics is. Because of the unquestioned inclusion of mathematics in the school curriculum, most teachers of the subject are not required to justify its place and purpose to pupils, parents or colleagues. Thus it is certainly the case that in most mathematics lessons pupils are taught rules, technical notation and established conventions of mathematics without acquiring any feeling for why the systems exist. However, when teachers have presented pupils with a broader mathematical challenge they have found them achieving more.

Some pupils' comments:

"I think I have achieved a lot more this year than any other year. Also I like maths a lot more now." (Set 7/7)" [1]

"I enjoyed maths the most this year because I had to really think about what I was doing and didn't just work out of books." (Set 4/5)

"In this type of group work we had to be really organized and co-operative with each other so we could get the times and lengths right and get the right results in our tables. Also it helped me to understand graphs and plotting graphs. We also came up with an explanation (conclusion) at the end of this experiment ... which was quite hard to write down because sometimes it is hard to put it into words ..." (Set 1/3)

"This year I have learned a lot ... but I think I've learned more about maths itself as I thought it was adding up but as you can see maths can be very practical." (Set 2/3)

"I have learned quite a bit this year. The most important one is how to work in a group. I've learned to listen and not interrupt others when they are talking. I have learned to wait patiently then ask questions after they have finished talking. I've got to know other people who I have never worked with before and to share my ideas with them and to explain them properly so they can understand them."

"I prefer it when we work in groups because it is better for morale and we felt we were being trusted to work. If I got stuck I tried to work it out for myself. If I couldn't I asked my friends or the teacher for help. I was able to explain some of the work I had done to other people and sometimes I wasn't able to because I managed to get myself totally tongue-tied." (Set 4/5)

"All this kind of work has given me a different attitude towards maths."

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1 Sets refer to ability groups.

The methods, materials, equipment and settings for learning

The teachers involved with the Project began to develop productive working links with bodies such as assessment agencies, parents, institutions of higher education and employers. Numerous successful strategies have been developed and documented by the Project, particularly with regard to parents and the community.

In order to establish a match between research and practice, action research methods of investigation were considered most suitable. This required the teacher-researchers and supporting teachers to keep personal logs and write case studies. This enabled them to provide a record of significant learning experiences that have taken place, to trial, reflect, review and refine their existing practices. For the Project's purposes this model is best defined as:

'A systematic study of attempts to change and improve educational practice by groups of participants by means of their own practical actions and by means of their own reflection upon the effects of those actions.'

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One of the outcomes of this process has been the documentation of 26 generalisable strategies for engaging pupils of all abilities in meaningful and productive mathematics.

Teachers and teacher education

The teachers' involvement in the Project had a marked effect on their own professional development. From our analysis of case studies, we have identified the following indicators which help describe teachers' professional development.

How teachers have:

- grown in confidence;
- found greater satisfaction in their classrooms;

- used existing structures to provide for pupils' changing needs, for example, evolving appropriate innovative assessment systems;
- taken on a wider view of professional responsibilities of their own choosing
  i. they have become more aware and involved in the work of the professional associations,
  ii. they have taken a leading role in activities such as parents evenings, in-service days, support groups,
  iii. they have demonstrated that they see curriculum development and research as an integral part of their job,
  iv. they have led workshops both in and out of school;
- not limited the work to their own classrooms;
- established links with other subject areas;
- become more discerning with regard to in-service provision, published material and other resources;
- set up support and working groups throughout the regions;
- actively involved parents and governors in the developments taking place which has led to an increased understanding and further support;
- contributed to a wide range of publications containing ideas, strategies and case studies.

"Participation is allowing us the time to ask questions such as 'What is maths?' ---- Teaching can be like driving in the fast lane of a motor way in that one has no time to think 'Is there a better way to go?' One can be compelled by the marking, the meetings, the preparation to continue on the same road. We are being allowed to consider some alternative routes on Thursday."

"Changes have been taking place in my classroom. My perception of how to teach mathematics was the teacher standing at the front of the room imparting knowledge for the benefit of his/her pupils followed by a series of exercises ---. Since I've been involved in RAMP I am beginning to question and challenge my own assumptions about the nature of mathematics and more practically the teaching of mathematics in my classroom. I no longer hold the somewhat conceited view that children learn mathematics because of my teaching."

"So what is different about our teaching methods? Well - we try to take time to listen to the pupils and accept their ideas; give them time by not expecting an outcome too soon; let them work in groups and use equipment as and when they want it; encourage the pupils to 'have a go' at maths and not to be afraid of tackling a problem; encourage the pupils to express themselves on paper, to each other and to the teacher; make the pupils more responsible for their own learning."

Assessment, evaluation and accountability

Amongst those concerned with the Project there were three separate but related groups of decision makers: the Project's participants; the Project's sponsors and the steering committee; interested outsiders. Each group required different information. Teachers were concerned to improve their own practice and establish future priorities. The sponsors were concerned with pedagogic issues but wanted to know about the Project's costs, use of resources and outside reputation. The outsiders wanted help in deciding whether or not the Project 'worked' or to see whether it could be applied or adapted to their own situation.

In order to address these needs a blend of internal and external procedures and personnel contributed to the evaluative process. The Project's aims and methodology required the team and the teachers to monitor and evaluate so that effective strategies could be formulated at every stage.

Teacher-researchers' case studies were collected, documented and analyzed. Since all these teachers worked within normal classroom conditions and not in experimental situations they were able to evaluate and assess outcomes within this context. The teachers evolved the following set of criteria for describing their pupils' progress in mathematics.

How pupils are showing improvement in:

- examination results (where applicable);
- test and assignment marks;
- attitude to mathematics i.e. showing enjoyment, perseverance, motivation, confidence and interest;

- willingness to take responsibility for organizing their own work in mathematics both within and outside school;
- ability to work cooperatively, discuss and write about mathematics;
- using practical equipment and technological devices;
- ability to apply their knowledge and skills in mathematics to other subjects and life in general;
- a willingness and interest to study mathematics beyond the age of 16.

The RAMP teacher release had been planned carefully on a long term basis enabling the schools to accommodate the teacher's absence and gain maximum benefit. Three of the most important factors this mode of release are:

- schools do not lose the expertise of the teacher-researchers;
- ideas and strategies developed over the three year period are shared with an increasing number of colleagues within their schools and LEAS;
- a self-sufficiency model which fosters a positive attitude and confidence in teachers in a society and an educational system which are both changing continually.

One head teacher described the RAMP model as:

"most cost effective and least disruptive".

SURVIVAL AND MATHEMATICS EDUCATION

Ed Jacobsen
UNESCO

If I were a secondary school mathematics teacher, I would think that the title referred to my survival in the profession, perhaps even for the day. A primary school teacher's survival would mean how to live on the salary paid. But I am going to use survival referring to students, that proportion of children enrolling and remaining enrolled in the school system, which is more commonly called retention. Those that don't survive are usually called drop-outs.

It is generally accepted that the acquisition and retention of literacy skills of a student is reached by grade 4 or 5 of primary school. What grade level would be necessary to guarantee numeracy (whatever that means)? I will propose some definitions later. After we define numeracy, can we say that after 4 years of schooling a student can function as a numerate adult? Six years?

UNESCO collects the world's educational statistics. Their calculations show that the survival rates in Latin America and the Caribbean are the lowest in the world, followed by sub-Saharan Africa. In Latin America and the Caribbean, of those initially enrolled, 74% reach grade 2, and only 60% attain grade 4. The highest rates are in the Arab States, with 97% reaching grade 2, and 92% grade 4. For Eastern Asia the figures are 94% and 86%, and for Southern Asia, 80% and 72%. Many highly industrialized countries have survival rates of 50% in their inner-city schools.

The above figures are only for drop-outs, and don't include those children who never began schooling. If we add the number of drop-outs to those who never were in school, we can calculate the percentage of the school aged population not in school, or *the excluded*. In sub-Saharan Africa this is half of all children! In Southern Asia, almost 40%, the Arab states 30%, and Eastern Asia 19%. (Fig. in appendix). The rates for individual countries are in Figure 2.

What does the future look like? What is the projection for out-of-school children of primary school-age? There are 110 million school-age children not in school, 90% of them living in Sub-Saharan Africa and Southern Asia, and those regions face a doubling of the population in 20 years because of growth rates of 3% in Africa and 2% in Asia. If we assume that the *proportion* of out-of-school to in school remains constant, the 3% birth rate in Africa will mean a 50% growth in enrollment in the next 12 years, and means the absolute numbers of out-of-school children going up by 50%. To maintain the absolute *number* of out-of-school children constant, 38M, enrollment will have to double in 12 years. For Latin America, corresponding enrollment increases are 24%. The graphs of this data with various assumptions are given as Figure 3.

There is a new concentrated attack on the provision of schooling for all children. UNESCO, UNICEF, and the World Bank convened a World Conference on Education for All in Jamtien, Thailand in 1991. Governments made commitments to provide education for all by the end of the century. However, already most countries spent about 25% of their budget on education, usually the biggest item after the military budget. Governments are having to reduce expenditures, and hence are asking education to be more accountable. Thus, many countries have recently evaluated the effects of schooling, and the results in mathematics are distressing. They show that most pupils after completing the primary school can carry out routine calculations in arithmetic, but cannot solve rather simple story problems requiring an application of mathematics.

(Of course, maths teachers get the criticism, as teachers are always the scapegoats of failures of national educational systems.) We can also find out what schools have accomplished in teaching mathematics by examining how adults use mathematics. You know of research done in both industrial and developing countries which shows that adults have very little recollection of school maths and limited problem-solving skills. Most adults avoid using mathematics, but when necessary use non-standard algorithms which they have invented themselves and which do not always work, rather than using methods which were supposedly learned in school. In the market-place, one finds few algorithms which a maths teacher would recognize. What is interesting with both adults (in the United Kingdom) and children (in Brazil) is that their answers using non-standard methods of solving problems in the market are usually correct or at least very close when these same people are given a

similar problem in a school context, the results are considerably worse, and completely unreasonable answers are not questioned.

What do we want our students to know in mathematics when they complete nine years of schooling? According to Professor James Fey, in the USA the most important goal in school mathematics is to develop students' ability to reason intelligently with qualitative information. In the United Kingdom the Cockcroft report defined numeracy as those mathematical skills that enable an individual to cope with the practical demands of everyday life. A major country in Asia wants its primary school leavers to be able to calculate rather complicated sums; ...in their heads.

Its important, this definition of numeracy, as it determines what should be done in schools. I hope we will have time to discuss this. What is the *real* goal of teaching maths in your country? Does the all determining school leaving examination support this goal, or does it force the teaching of a different one? Does the maths programme just prepare all students for the next level? If so, are the mathematical needs of those pupils for whom primary education is terminal compatible with those of pupils who will continue to higher levels of schooling? In other words, can one mathematics programme serve both those children who drop out of formal education at an early age, and those who continue their education and ultimately make up the nation's higher level work force? For the nine years of compulsory school, I believe that the answer to this last question is yes, providing mathematics learning is closely linked with realistic problem solving. There are many reasons for teaching mathematics from situations familiar to students. One is that motivation is higher when pupils know that what they learn can be applied. Researchers from many countries have recommended that girls need a more active, experimental teaching approach to overcome their passive learning of mathematics. Another reason is the possibility of catering for urban/rural differences by choosing problems appropriate to each in teaching the same mathematical processes. Hence, students in a rural school, those in a village, and others in the city, all learn the same mathematical concept using situations appropriate to their own communities. Technology opens entirely new approaches to thinking about mathematical ideas and problems. Even before the advent of microcomputers, the proliferation of pocket calculators had made the skill of carrying out long calculations on paper obsolete, and students who have only learnt to carry out

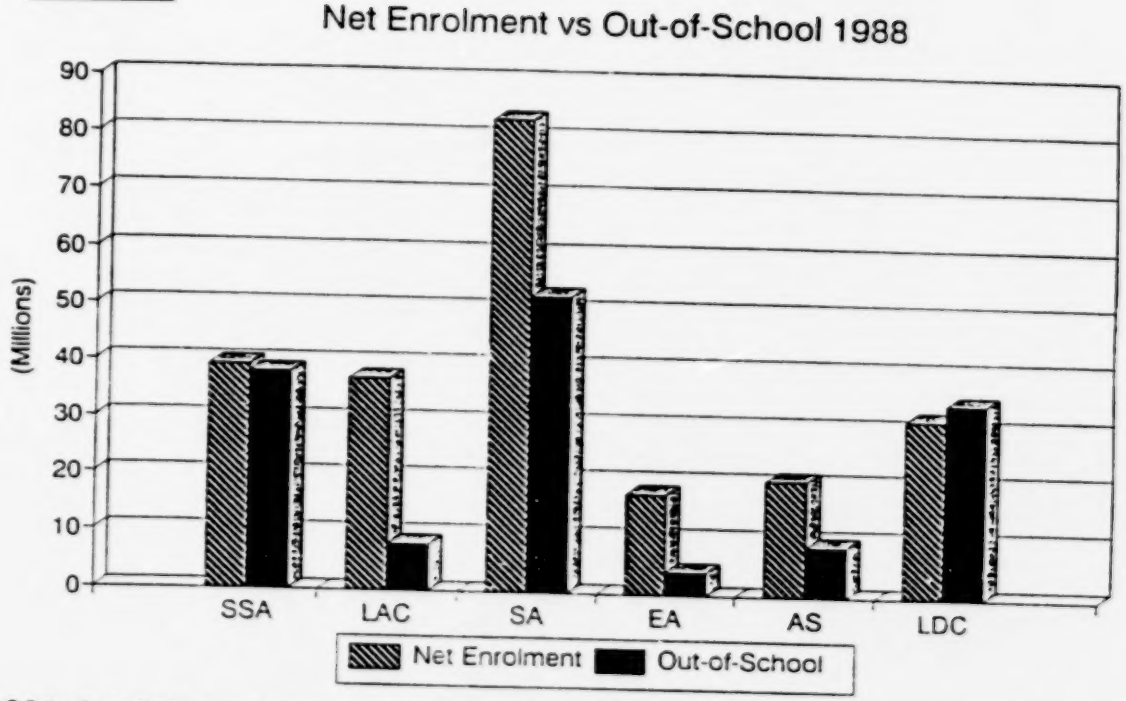
arithmetical calculations in their many years of primary school end up with no more skills than can be found in a pocket calculator costing the price of two cups of coffee. The ubiquitous calculator! To be found in most countries wherever business is carried out, and in the school bags of most children in the hopes that their teachers will allow them to be used. Thus there should be a shift of emphasis away from speed and accuracy of paper and pencil algorithms to conceptual understanding and problem-solving strategies which will allow people to use calculators effectively. A major task is to find a balance between conventional skills, understanding, and problem-solving ability appropriate to the new technological environment.

I want to digress for a moment, to tell you about the next book in my UNESCO series *Studies in Mathematics Education*. Volume 10 will be devoted to the teaching of numeracy. I hope this Working Group will give me ideas for the book. In addition, there will be an invitational Conference on numeracy, in Paris (not convened by UNESCO) 17-22 October 1992, and I am hoping this meeting will help to identify possible contributors.

We were asked to consider why "they" drop out? A simple answer for many; because they must earn money for the family, or they wish to buy things for themselves. I think in many countries language is a big factor for school dropout. In almost all countries of the world, a substantial proportion of pupils (in many countries, all of them) are schooled in a language other than their mother tongue. Then to make things even more difficult, they have to master the specialized languages and symbolism of mathematics and science. By learning mathematics through already familiar life situations, the pupils replace gradually the terminology of their mother tongue with the symbols and language of mathematics, rather than being forced to learn this specialized language all at once. Many countries in Africa now give instruction in the mother tongue at least for the crucial initial years of primary school.

It is my hope that during the three meetings of the Working Group, we can arrive at some suggestions that those responsible for mathematics education can consider and possibly implement.

Figure 1



SSA: Sub-Saharan Africa LAC: Latin America and Caribbean SA: Southern Asia

EA: Eastern Asia and Oceania AS: Arab States LDC: Least Developed Countries

Figure 2: Primary School Age Population

1988 Enrolled and out-of-shool children (in thousands and as percentage)

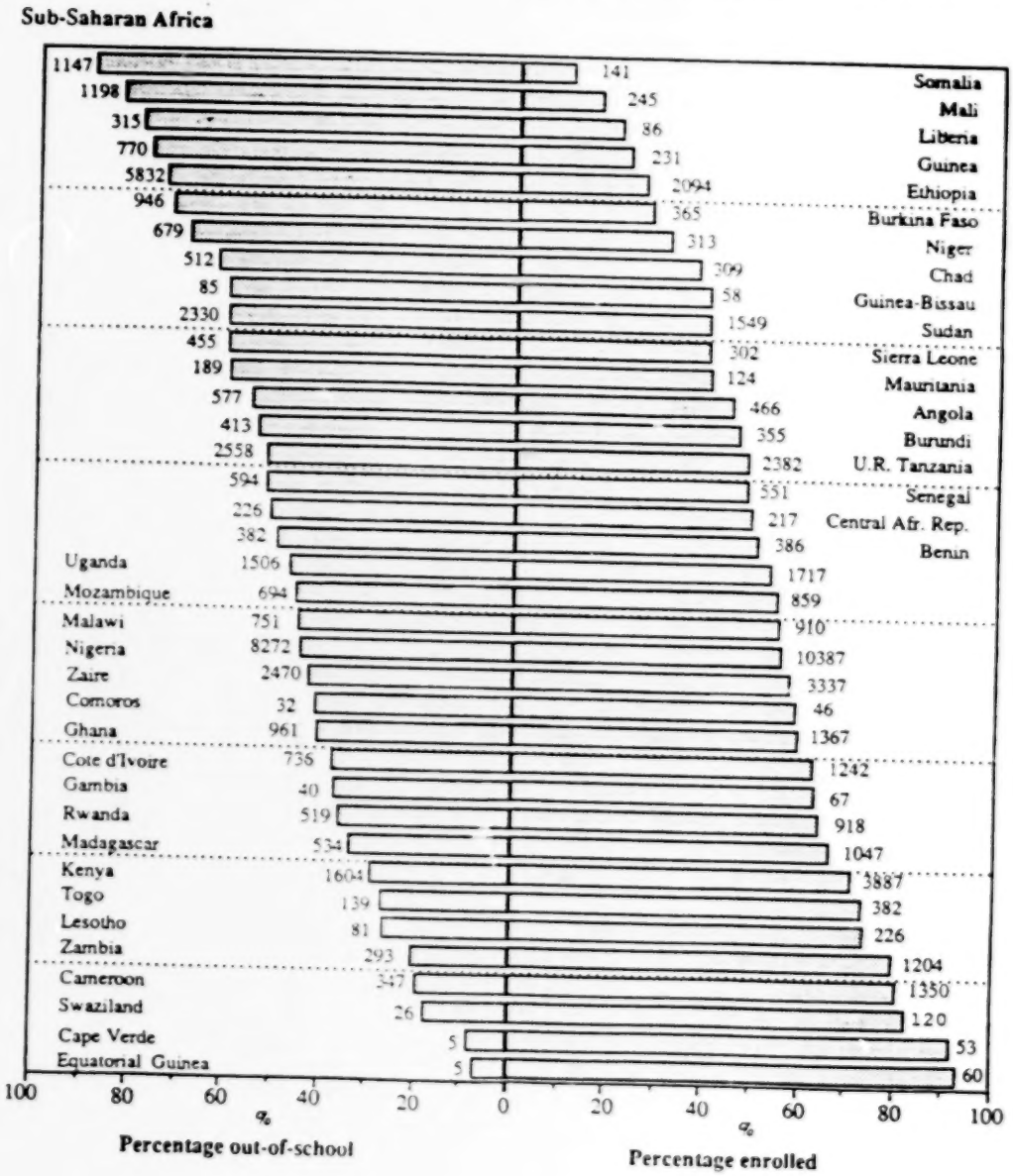
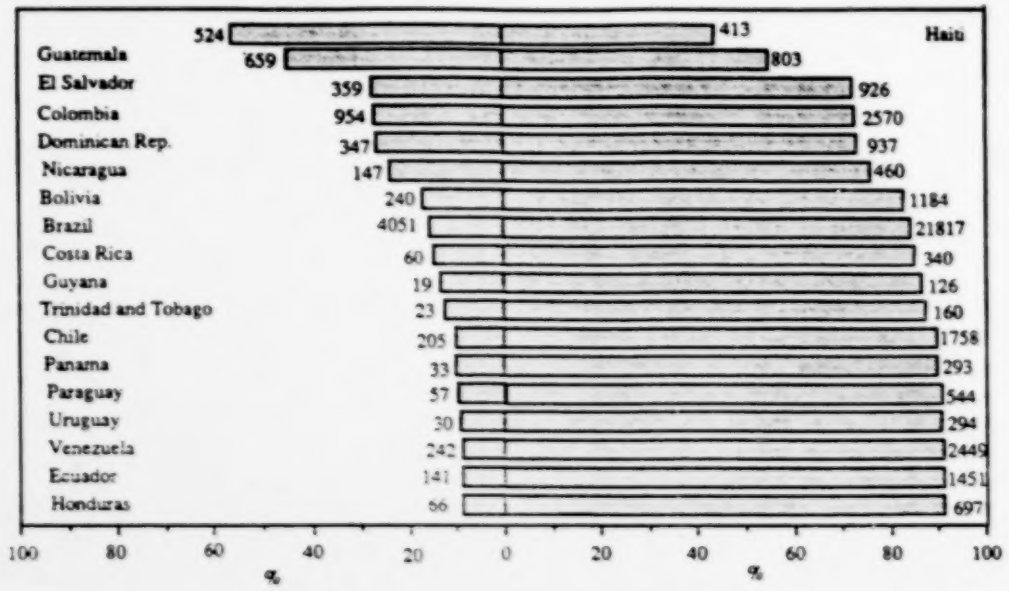


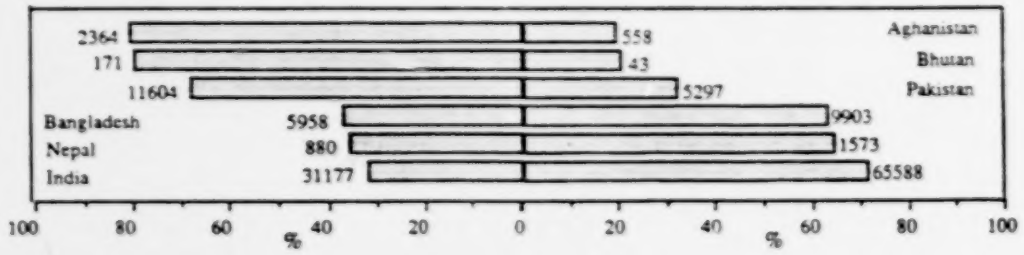
Figure 2 (cont'd): Primary School Age Population

1988 Enrolled and out-of-school children (in thousands and as percentages)

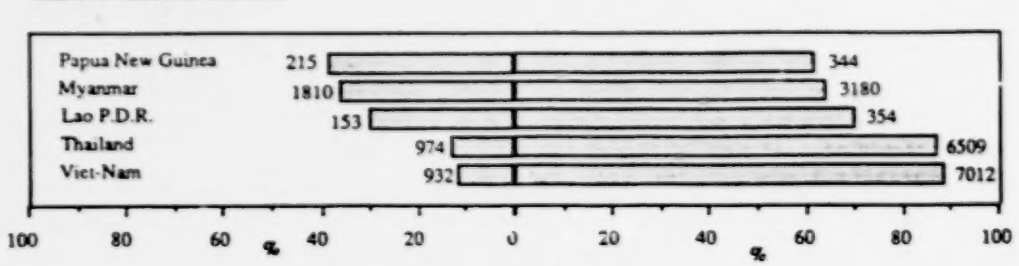
Latin America and the Caribbean



Southern Asia



Eastern Asia and Oceania



Percentage out-of-school **Percentage enrolled**

Figure 2 (cont'd): Primary School Age Population

1988 Enrolled and out-of-school children (in thousands and as percentages)

Arab States

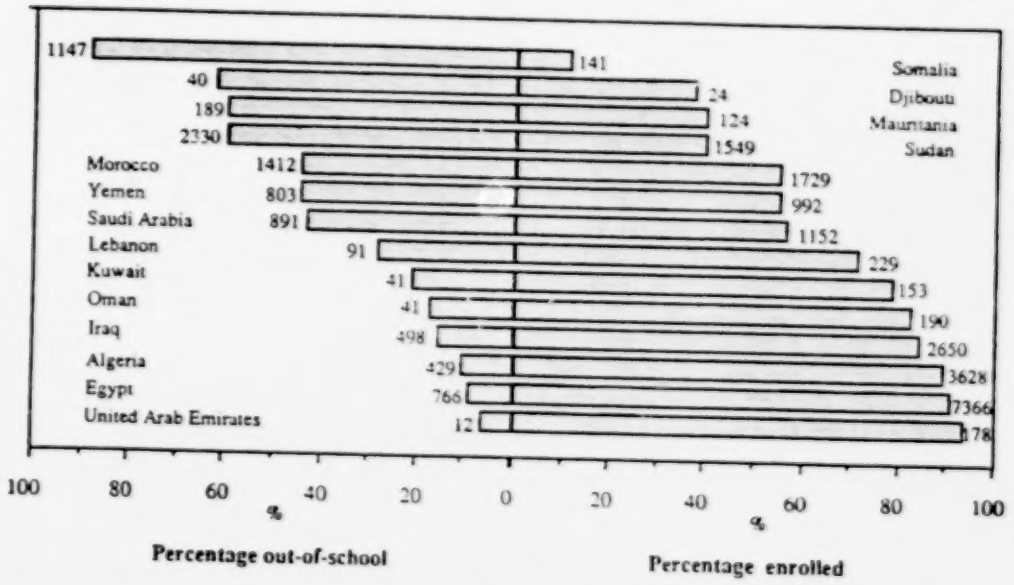
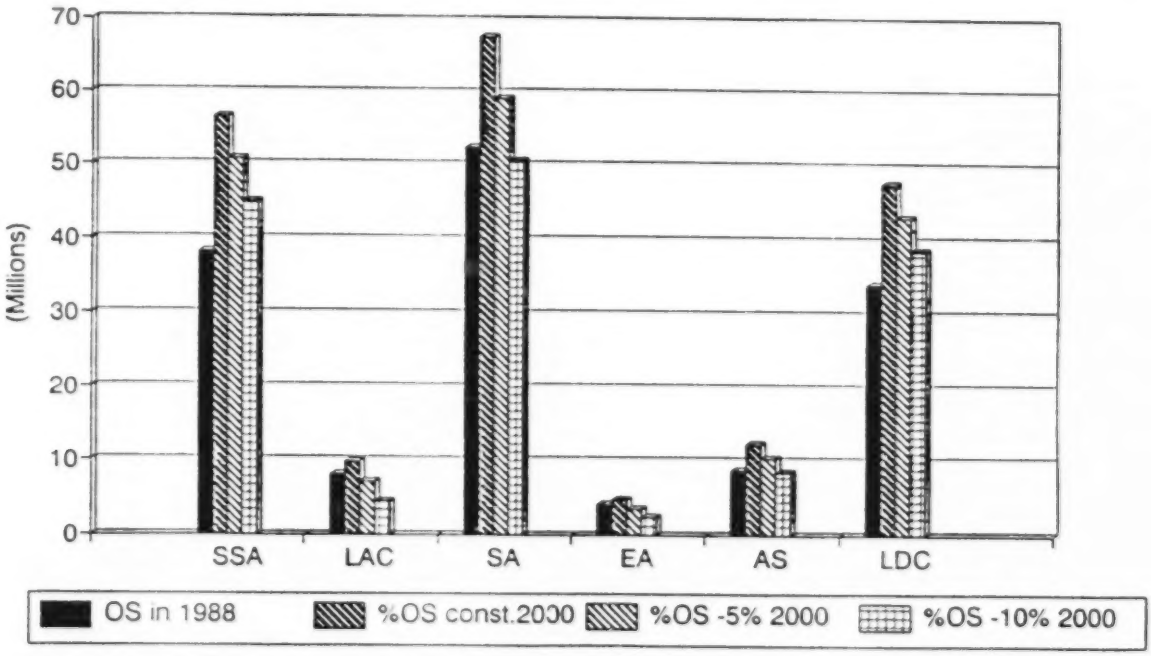


Figure 3

Out-of-School (OS) 1988 and 2000



SSA: Sub-Saharan Africa LAC: Latin America and Caribbean SA: Southern Asia
EA: Eastern Asia and Oceania AS:Arab States LDC: Least Developed Countries

BASIC NUMERACY FOR SCHOOL LEARNING

Anthula Natsoulas
University of Toledo

Children have always gone to school to learn the three R's -- "reading, 'riting, and 'rithmetic," the traditional components of a minimal education. Often, learning 'rithmetic consisted of completing pages of drill exercises in addition, subtraction, multiplication and division. Today's children still go to school to learn basic skills but the numeracy skills which they will be taught must be viewed within a broader context. In trying to define basic levels of mathematical literacy, we must look to the role of mathematics in daily life and the mathematical needs of the future.We are currently undergoing major changes paralleling those of the industrial revolution; we are moving into an information age dominated by a new kind of technology. The rapid growth of knowledge makes it difficult for us to predict in what kind of world today's children will reach adulthood. We can no longer prepare children for the future by teaching them to live in the present. Today's youth must be prepared to function in as yet an unknown environment and to solve problems not yet formulated. It has thus become even more imperative that young people achieve an acceptable level of mathematical literacy. This paper will address the question of what comprises basic numeracy.

Numeracy has been defined as referring to "those skills of numerical and spatial thinking required for daily living in society" (Brinkworth, 1985, p. 8). Alternately, "informed numeracy" has been defined as knowing: (i) when to use mathematics; (ii) what mathematics to use; (iii) how to do mathematics, and (iv) how to use the results provided by the mathematics" (Open University Course Team, cited in Chapman and Lee, 1990, p. 278). It would perhaps be more productive to include numeracy within a broader framework of basic literacy, erasing the distinction between literacy and numeracy. In this context, the goal of schooling would be to provide students the basic skills which would prepare them to be fully participating members of a highly technological society. The resultant prescriptions for minimal levels of school mathematics should be made within the

context of this broader conceptualization of numeracy and with the recognition of changing societal and educational expectations:

1. A recognition that the types of mathematical thought which the citizen of the future will need to function in daily life is changing.
2. Increased demands by the workplace for more mathematical knowledge by a larger segment of the population.
3. Concurrently the need for a different way to teach mathematics because of the presence of calculators and computers.

In developing a description of minimal levels of school mathematics, two broad questions must be considered: (1) why should a student learn mathematics and (2) what are the minimal levels of numeracy. In response to the first question, it is proposed that the answer falls into three categories:

1. The acquisition of career-related skills.
2. The acquisition of attitudes and knowledge which will result in an informed member of society.
3. The acquisition of attitudes, knowledge and values which will be part of an educated citizen. (Chapman and Lee, 1990; Evans, 1989; Paulos, 1988).

In response to the second question, there are a number of sources which can be consulted in formulating the parameters for basic levels of mathematical learning: (1) the activities of daily life in a complex technological society, (2) reports about and recommendations for the current and future state of school mathematics, (3) major studies of school mathematics, and (4) items included on proficiency and achievement tests.

Young people will enter the job world directly from their secondary schooling or go on to college. For those who enter the job world, there are specific mathematical skills which will be needed as evidenced by basic skills tests given by prospective employers (i.e., civil service exams). At the very least, algebra and geometry have been identified as minimal background to provide access to further training as well as preparation for basic skills tests. If such basic levels are not met, many jobs may be beyond the reach of an

applicant. For those students who go on to post-secondary training either immediately upon leaving secondary school or return later in life, it will be found that many courses of studies require higher levels of mathematics. Beyond minimal numeracy, a basic proficiency level is necessary so that career options will not be seriously limited (Evans, 1989; Paulos, 1992; Sells, 1980).

The need for a minimal mathematical background to form a foundation for the development of job skills is obvious. Equally necessary, however, is the need for a basic level of numeracy to allow full participation in today's complex world. "Drawing inferences, perceiving logical relationships, evaluating cause and effect, ordering and sequencing . . ." (Chapman and Lee, 1990, p. 283) in all areas of life require a degree of mathematical sophistication. Within the school curriculum there is the need to read symbolic, graphical, tabular or diagrammatic text. On a daily basis we are constantly bombarded with all kinds of mathematical data. Such information may refer to rates of inflation or unemployment, the results of gallup polls, sale prices, differing return rates on savings accounts, interest rates on loans, risk factors in developing a particular disease. Basic numeracy skills are needed in order to adequately understand and assimilate such information and to be prepared to make informed decisions. Inadequate understanding of mathematical principles not only restricts future career options but can lead to the acceptance of false claims and vulnerability to hoaxes (Sells, 1980; also see Paulos, 1988).

Thus, mathematical knowledge must not be the province of the few. The minimal levels of numeracy to be described for all students must go beyond basic arithmetic. The traditional school mathematics curriculum which has emphasized arithmetic, measurement, algebra, and geometry presented in a hierarchical sequence and most of which is more than 300 years old is no longer appropriate. In trying to develop a new definition for mathematical literacy the content of school mathematics must be critically examined to see whether it responds to current needs and whether topics from newer branches of mathematics which may be valuable. Suggestions include the addition of informal logic, analytic geometry, combinations, probability, graph theory and game theory to the school curriculum. In addition, the very conception of basic learning must be revised to include the acquisition of process skills such as understanding problems, implementing strategies for solution, interpreting data, transferring skills to new problems, tackling unseen problems,

using critical judgment, working positively in groups as well as a teaching approach which provides informal experiences emphasizing the patterns of mathematics. Furthermore, as the routine use of calculators and computers becomes more commonplace, the use of estimation skills to assess the reasonableness of answers increases in importance (Burghes, 1989; Edwards, 1984; Paulos, 1988; Steen, 1990).

Two major reports within the past decade which have made recommendations for the teaching of mathematics are the Cockcroft Report in England (Committee, 1982) and the Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics (NCTM Standards) in the United States (Commission, 1987). In studying these two documents, it is striking how a number of common elements emerge:

1. Use of concrete experiences in the early years.
2. Development of mathematical language.
3. Development of problem solving strategies.
4. Content core of mathematics learning for all students.
5. Expanded world view of the role of mathematics.
6. Integration of calculators and computers at all levels.

Specifically, the NCTM Standards identify common strands across all grade levels which may be considered as process skills: mathematics as problem solving, mathematics as communication, mathematics as reasoning, and mathematical connections. The common content strands common across all grade levels include: basic operations (whole numbers, fractions and decimals), statistics and probability, geometry, and algebra. Included, in addition, at the primary levels are: estimation, measurement, patterns and relationships and at the middle grades levels: number, number relationships, number systems and number theory, computation and estimation, patterns and functions, measurement. At the senior high level more advanced topics are included such as functions, discrete mathematics, trigonometry, mathematical structure, conceptual underpinnings of calculus (Commission, 1987).

There a number of major studies at both national and international levels. The International Studies include the First International Mathematics Study (FIMS) in 1967, the

Second International Mathematics Study (SIMS) in 1981/82 and the Third International Mathematics Study (TIMS) currently in the piloting stage. These have been conducted under the coordinating efforts of the International Association for the Evaluation of Educational Achievement (IEA). In the case of SIMS, the test were administered to two levels of students: Population A (all students in the grade level where most had reach the age of 13.00-13.11 years by the middle of the school year) and Population B (all students in the normally accepted terminal grade of secondary schooling and who are studying mathematics as a substantial part of their program). For the purpose of defining levels of mathematical literacy, at the very least those items given to Population A should be considered.

The Population A categories for SIMS included:

- Arithmetic (62 items): whole numbers, fractions, decimals, ratio/proportion, percent, exponents, square root.
- Algebra (32 items): integers, formulae/expressions, equations, exponents, sets.
- Geometry (42 items): figures/proportions, congruence, similarity, Pythagorean relation, coordinates.
- Measurement (26 items): metrics, linear scales, estimation, area, volume.
- Statistics (18 items): average, tables/graphs. (McKnight, et al., 1990; United States National Coordinating Center, 1986).

Considering items in the pilot tests of TIMS, parallel content strands may be found. In the case of the newest study, we see an emphasis on problem solving, applications/connections, reading interpreting mathematics, estimation, and even a question which requires students to solve a mathematical problem in a cooperative environment (Test Preparation Committee, 1991).

The National Assessment of Educational Progress (NAEP) in the United States has been given periodically to third, seventh and eleventh grade students in the United States since 1978 and a fifth testing has just been completed. In its later studies, the NAEP defines both content and process categories:

- Content categories: fundamental methods of mathematics, discrete mathematics, data organization and interpretation, measurement, geometry, relations/functions and algebraic expressions, number and operations, calculators.
- Process levels: knowledge, skill, understanding/comprehension, routine application, problem solving/reasoning. (Lindquist, 1989)

Furthermore, in the United States, because of the increased concerns about the inadequate levels of mathematical preparation of our students, various states have implemented proficiency tests which define minimal levels of attainment as requirements for graduation from high school. In the state of Ohio, at the ninth grade level, both content and process strands are identified:

- Content strands: arithmetic, measurement, geometry, data analysis, algebra
- Process levels: knowledge and skills, conceptual understanding, application and problem solving.

In summarizing the recommendations and responding to the stated needs and considering the content of the various sources which were consulted, the shape of school mathematics becomes more clearly defined. Numeracy must be conceptualized to include the development of reading and comprehension skills applied across the curriculum and in daily life. A mathematically literate person should be one who is empowered to respond to the demands of a highly technological society. Three basic assumptions should underlie all mathematics learning:

(1) there is a minimum mathematical common core for all students;
(2) calculators and computers are an integral part of the mathematics classroom;
(3) and applications to and connections with other areas of life and the school curriculum are implicit.

Two categories for learning can be identified: process skills and content strands. Under process skills are included:

(1) an experiential approach to teaching and learning mathematics,
(2) the development of problem solving strategies,
(3) the use of a cooperative learning environment,
(4) the development of mathematical reasoning.

The content strands should include:

(1) basic arithmetic operations with whole numbers, fractions, percent, decimals, including an understanding of ratio and proportion,
(2) estimation skills,
(3) basic geometric concepts,
(4) probability and data analysis,
(5) basic algebraic skills.

The development of numeracy skills is just as important as learning to read and write. A view of mathematics as more than basic manipulation of symbols and memorization of facts is necessary in preparing young people for the next century. The traditional "reading, 'riting, and 'rithmetic" skills must be reconfigured to reflect a changing world and in this new view be seen as a basic right for all children.

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EQUITY AND EQUALITY: WE MUST DO MORE THAN MOUTH THE WORDS

Iris Carl
Houston, Texas

Definitions of numeracy:

"Those skills of numerical and spatial thinking required for daily living in society."

Alternately, 'informed numeracy' has been defined as knowing:

(i) when to use mathematics;
(ii) what mathematics to use;
(iii) how to do mathematics, and
(iv) how to use the results provided by the mathematics.

Underlying assumptions for school mathematics:

1. A recognition that the types of mathematical thought which the citizen of the future will need to function in daily life is changing.
2. Increased demands by the workplace for more mathematical knowledge by a larger segment of the population.
3. Concurrently the need for a different way to teach mathematics because of the presence of calculators and computers.

Two questions to be considered in developing minimal levels of school mathematics:

(1) Why should a student learn mathematics and
(2) what are the minimal levels of numeracy?

Answers:

Why:

1. The acquisition of career-related skills.
2. The acquisition of attitudes and knowledge which will result in an informed member of society.
3. The acquisition of attitudes, knowledge and values which will be part of an educated citizen.

Sources for the answer to 'what':

1. the activities of daily life in a complex technological society,
2. reports about and recommendations for the current and future state of school mathematics,
3. major studies of school mathematics, and
4. items included on proficiency and achievement tests.

Major reports: The Cockcroft report in England and the curriculum and evaluation standards of the National Council of Teachers of Mathematics in the United States.

Common elements:

1. Use of concrete experiences in the early years.
2. Development of mathematical language.
3. Development of problem solving strategies.
4. Content core of mathematics learning for all students.
5. Expanded world view of the role of mathematics.
6. Integration of calculators and computers at all levels.

Specific elements of the NCTM Standards:

Specific strands identified by the NCTM Standards across all grade levels include:

- mathematics as problem solving;
- mathematics as communication;
- mathematics as reasoning, and
- mathematical connections.

The common content strands common across all grade levels include:

- Basic operations (whole numbers, fractions and decimals),
- statistics and probability,
- geometry,
- and algebra.

Included, in addition, at the primary levels are: estimation, measurement, patterns and relationships and at the middle grade level: number, number relationships, number systems and number theory, computation and estimation, patterns and functions, measurement.

At the senior high level more advanced topics are included such as functions, discrete mathematics, trigonometry, mathematical structure, conceptual underpinnings of calculus.

International studies:

The Population A categories for SIMS included:

Arithmetic (62 items):	whole numbers, fractions, decimals, ratio/proportion, percent, exponents, square root.
Algebra (32 items):	integers, formulæ/expressions, equations, exponents, sets.

Geometry (42 items):	figures/proportions, congruence, similarity, Pythagorean, relation, coordinates.
Measurement (26 items):	metrics, linear scales, estimation, area, volume.
Statistics (18 items):	average, tables/graphs.

Parallel strands on the TIMS pilot tests: an emphasis on problem solving, applications/connections, reading interpreting mathematics, estimation, and a question which requires students to solve a mathematical problem in a cooperative environment.

The United States

National achievement testing

The National Assessment of Educational Progress (NAEP) in the United States has been given periodically to third, seventh and eleventh grade students in the United States since 1978. In its later studies, the NAEP defines both content and process categories.

Content categories: fundamental methods of mathematics, discrete mathematics, data organization and interpretation, measurement, geometry, relations/functions and algebraic expressions, number and operations, calculators.

Process levels: knowledge, skill, understanding/comprehension, routine application, problem solving/reasoning (Lindquist, 1989).

State proficiency testing (Ohio):

Content strands: arithmetic, measurement, geometry, data analysis, algebra.

Process levels: knowledge and skills, conceptual understanding, application and problem solving.

Summary and recommendations

Assumptions for numeracy:

Numeracy must be conceptualized to include the development of reading and comprehension skills applied across the curriculum and in daily life.

A mathematically literate person should be one who is empowered to respond to the demands of a highly technological society.

Assumptions for mathematics learnings:

(1) There is a minimum mathematical common core for all students,
(2) calculators and computers are an integral part of the mathematics classroom and
(3) applications to and connections with other areas of life and the school curriculum are implicit.

Categories for school mathematics: process skills and content strands

Under process skills are included:

(1) an experiential approach to teaching and learning mathematics,
(2) the development of problem solving strategies,
(3) the use of a cooperative learning environment,
(4) the development of mathematical reasoning.

The content strands should include:

(1) basic arithmetic operations with whole numbers, fractions, percent, decimals, including an understanding of ratio and proportion,
(2) estimation skills,
(3) basic geometric concepts,
(4) probability and data analysis,
(5) basic algebraic skills.

MATHEMATICS FOR PREMATURE SCHOOL-LEAVERS

Dina Granove
Winnipeg, Manitoba

The Winnipeg Adult Education Centre opened in 1966 to provide adults the opportunity to obtain a high school certificate. Since then, the enrollment has expanded along with the program to suit the various needs of the adult population. The school operates two five-month terms per year with courses offered during the daytime as well as in the evenings. There are presently over 1300 students enrolled in the day school and over 850 in the night school. Most of these students have had previous difficulties with school; the majority have children and are single parents; most have personal and/or financial problems; and many have been displaced from their land of birth.

The Centre offers a Basic Literacy component intended for adults with limited skills in reading, writing and numeracy. Generally these students are economically and socially disadvantaged and in some cases limited in their ability to learn. The students are assessed upon application to the Basic Program and are suitably placed in an individualized program of studies. A math component is added once they have attained about a grade four reading level. This math program is intended to give the students a better understanding of numbers and to equip them with the skills to compute whole numbers, simple fractions and decimals. The intent is for these students to proceed to the 090 (grade nine equivalence) level. For those students who are limited in their abilities and for whom it becomes apparent that moving on into the 090 level is unlikely, materials have been developed to enable them to become numerically functional in life with a calculator. The topics include money matters such as making change, banking skills, everyday percentage problems, etc.

The 090 level involves a five-month program of studies in English, Mathematics, Science and Social Studies. The 090 Math course is intended to solidify number skills in whole numbers, fractions, decimals, percents and signed numbers. The purpose of the program is to prepare the students for the 190 (grade ten equivalence) level into which

many of the students proceed. Some, however, leave at this stage for employment or training which requires only this level of education.

The 190 level is also a five-month program with courses again in English, Mathematics, Science and Social Studies. The 190 Math course is the level of introduction to algebra involving the following topics: review of signed numbers including whole numbers, fractions and decimals; polynomials; linear equations and inequalities in one unknown; introduction to the co-ordinate system; minimal factoring of polynomials; and basic geometry. It is at this point that students continuing onto higher levels at the centre need to make a choice as to their next mathematics course. The "non-mathematical" students can choose at this point to take the Math 301 course for a "general" grade twelve math credit. It is a consumer oriented math program involving limited algebra; equation solving (linear and quadratic); taxation and insurance; trigonometry; graphing and variation; permutations, combinations and statistics; mortgages; and budgeting. Those students who wish to proceed instead towards Math 300, a "university level" grade twelve math course, must first take the Math 290 course in their grade eleven program of studies.

The 290 level requires study in English, Mathematics, and two of Social Studies, Biology, Physics or Chemistry. Once again the Mathematics studies may be at the 301 level or the 290 level. The 290 program involves: polynomials and exponents; radicals; factoring and rational expressions; linear equations and inequalities in one unknown; quadratic equations in one unknown; an introduction to co-ordinate geometry involving transformations, distance and slope; trigonometry of right triangles; relations and functions; linear relations; and quadratic functions.

In order to graduate as a mature student from the Winnipeg Adult Education Centre students must obtain six grade twelve credits. The centre offers such credits in English (300– two credits, 301/305, Visual Communications 305), Mathematics (300/301), Science (Physics 300, Chemistry 300, Biology 300), Social Studies (History 300/301, Geography 300, World Issues 300, Economics 301, Law 302) and Business Education (Typewriting 102/202/302, Accounting 202, Office Procedures 302). All of these course follow the curricula as laid out by the provincial department of education. Students select the combinations of courses as required by the post-secondary institutions into which they seek

entry. As Mathematics 300 is one of the entrance requirements for many post-secondary programs, a majority of the students opt to take Math 300 as one of their grade twelve credits. The 300 program in Math covers the following topics: Polynomial and Rational Functions; Trigonometry and Circular Functions including Sinusoidal Functions; Analytic Geometry and Conics in Standard Position; Exponential and Logarithmic Functions; Sequences and Series; and Binomial Expansion Theorem.

As stated earlier, many students leave the centre once they attain their 090, 190, or 290 certificates for employment purposes or to enter training programs requiring grade 9, 10 or 11 equivalents. Others may leave after attaining specific grade twelve credits needed to proceed with their future plans. However, over the past five years the center has graduated over 150 students per year from the day school. Almost all of these graduates apply for further study at either of the two local universities or the local community college. Others enter smaller post-secondary programs such as the various schools of nursing operated by teaching hospitals.

The people who attend the Winnipeg Adult Education Centre have one basic thing in common — they want to affect a change in their lives and they believe that furthering their education is the answer. Many of them want to impact the importance of education on their children and by becoming adult students they can assist their children in their schooling as well as provide good role models for them. Consequently, the Winnipeg Adult Education Center is providing an invaluable service to the community, one with long term consequences.

INTEGRATING NUMERACY INTO THE LITERACY CURRICULUM

Tom Ciancone
Toronto, Ontario

The need for numeracy instruction

When I asked one of my students why she wanted to study math, she said, "When I go shopping, I want to know when people cheat me." Numeracy has been defined by Cockcroft (1982) as "an ability to cope confidently with the mathematical demands of everyday life." So when a person in our society lacks this ability, she or he is being cheated every day. Numeracy is a basic need and everyone has the right to have it.

Not much hard research has been done in Canada in the area of adult numeracy, but there are some relevant statistics that we can garner from the 1989 Survey of Literacy Skills Used in Daily Activities (National Literacy Secretariat, 1990). The survey consisted of interviews administered to individuals in their homes and involved a series of tasks designed to test reading, writing and numeracy abilities with respect to activities commonly encountered in daily life in Canada. The following results come from a sample of 9, 500 persons aged 16 to 69:

- 62% Canadians who have the numeracy skills sufficient to handle the numerical tasks normally encountered in every day life.
- 24% Canadians who can deal with commonly encountered documents and forms requiring them to perform simple numerical operations such as addition or subtraction.
- 14% Canadians who have very limited numeracy abilities which enable them to, at most, locate and recognize numbers in isolation or in short texts.

Further analysis showed that the 12% of adult Canadians with lowest reading skills also have limited numeracy skills.

These statistics, taken at face value, point to a serious problem of innumeracy among adult Canadians. Furthermore, my own personal experience and that of my many colleagues working in adult basic education suggest that provision of numeracy instruction with an appropriate methodology is a compelling need.

Mathematics as a communication skill

Mathematics is not a language but it is a tool of communication. The ability to use this tool provides a person the opportunity to express facts, opinions and to analyze things in the real world. Knowing how to calculate percentages, for example, is necessary for discount shopping and for understanding statistics on unemployment or free trade. For an adult to acquire math skills is not necessarily an easy task. "The power, and difficulty, of mathematics lie in its abstract nature and the fact that, unlike language, much of it is not used continuously in everyday life." (Penney, 1984:22)

To convey the abstract concepts, mathematics uses a code. The code of mathematics, its symbols and operations, form part of our English language. This code is not as flexible as a language, with its dialects and slang, because it must be standardized so that everyone can understand it. For example, "2 + 2 = 4" is almost universally understood.

In spite of its universality, the code of mathematics must be learnt through a language. Among other things, a student's weakness in numeracy may be based on his/her limitation in using language. For this reason, it can be beneficial for a learner to work at acquiring numeracy and literacy skills together.

Toward a numeracy methodology

In developing a methodology for numeracy instruction, we must consider the nature of mathematics learning, as we began to explore above, but also the nature of the adult learner. Determining appropriate instructional methods will depend both on the learner's current mathematical skills and on his or her attitude towards the use of mathematics.

Traditionally mathematics has been taught as a linear process with each of the four operations presented separately. From this approach, learners develop mechanical skills, but may lack the ability to name what they are doing, to estimate answers, to decide which operation is needed or to apply the skills in daily life.

In a more learner-centered setting, literacy instructors often use a functional approach by choosing themes such as budgeting, shopping, recipes or travel. While avoiding the boredom of traditional math, this approach runs the risk that gaps in mathematical understanding are missed in the absence of a structure. In A *Sequencing Guide for Numeracy: Whole Numbers* (Lucas, Dondertman & Ciancone, 1991), we propose a spiral approach for numeracy instruction which provides a framework for skill-building while addressing students' immediate needs.

Whatever approach we use, it must start with the learner's present knowledge and build from there. There should be a balance between functional needs and skill-building. The learner should be encouraged to continually make connections between the mathematical operations themselves and the applications in daily life. Since the world of mathematics has all kinds of games and puzzles, a numeracy lesson can also be fun.

Facilitating numeracy

"Literacy instructors are ideally suited to facilitate numeracy activities. They know how to integrate real life situations into the content of their teaching, and they have the experience in working with students of varying backgrounds and skill levels. The attitudes of the instructor as well as the atmosphere in the classroom can encourage learners to address their numeracy needs in the context of their literacy needs. Through participatory activities, instructors and students can become comfortable with numbers. In fact, the experience in facilitating literacy that instructors bring to their practice is vital in bridging the gap between literacy and mathematics." (Lucas, Dondertman & Ciancone, 1991)

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IMPROVING THE TRANSFERABLE SKILLS FOR TOMORROW'S WORKFORCE

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Materials, resources and teaching approaches which are designed to aid in the improvement of numerical skills have tended to concentrate on skills for tackling immediate problems and have not addressed the tension between focusing on specific applications and thereby gaining motivation on the one hand and the probable lack of transfer on the other. This paper raises some issues for consideration by curriculum developers, school superintendents, principals and researchers.

1. Technological changes, especially in the means of production and communication, have raised the skill levels needed for employment and transformed the structure and management of organizations. The dramatic rate at which change is taking place has made the notion of job permanence obsolete. The emphasis has moved to outcomes such as 'transferable' skills, adaptability, positive attitudes to change, the ability to communicate, desirable personal and interpersonal qualities and problem solving capabilities. One consequence is that mathematics curricula are increasingly incorporating practical applications. The Mathematics Centre's work[2] in this area is one example of an attempt to tackle this 'head-on' with the realization that the availability of calculators, computers and associated software is making possible the achievement of these new skills within mathematics.

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2 Publications associated with the following Research and Development Projects based at The Mathematics Centre, West Sussex Institute, contain full details of salient issues outlined in this paper along with strategies of teaching approaches. A list of these can be obtained from The Mathematics Centre.

1. Low Attainers in Mathematics Project (LAMP).
2. Raising Achievement in Mathematics Project (RAMP).
3. Teaching, Evaluating and Assessing Mathematics (TEAM).
4. Flexible Approaches in Learning Mathematics (FLP).

2. Countries throughout the world are now moving towards a more vocational approach to general education and a more general approach to vocational education with the acknowledgment that focusing on specific skills will not achieve the desired outcomes such as practical capability and the ability to use knowledge and skills as resources in solving real-world problems. These are the very aspects which will enable the vocational/academic routes to converge.

3. Mathematics courses should act as a foundation which enables students to embark on a variety of courses and training programmes, to undertake their own further study, and to follow a career/job route which may include study. It is recognized that in order to improve the academic standards of all young people there must be opportunities which offer a broad preparation for employment as well as an accepted route to higher qualifications and opportunities to study for vocational qualifications without restricting their future choices.

4. To design separate mathematics courses for each vocational qualification would be impracticable. It would be more helpful to group vocational areas which have comparable mathematical needs and then provide appropriate contexts for students to be able to recognize and apply mathematical techniques and make effective use of technology. This approach would address the tension between focusing on specific application and thereby gaining motivation on the one hand and the possible or probable lack of transferability on the other.

5. Since there cannot be standard entry requirements to vocational courses, it is not only essential to have good diagnostic tools for identifying the gaps in knowledge and understanding for individual students but also to have better diagnostic procedures for analyzing students' potential. This will help prevent wastage and enable suitable individual action plans to be drawn up. The move in the UK towards reducing some of the vocational courses from 3 years to 2 years, combined with the 80% workshop and 20% classroom-based study, will lead inevitably towards the need to improve back-up provision.

6. Teaching materials must include applications of mathematics drawn from practical contexts which enable students to work successfully in their chosen or a related field of study. The necessity to cater for progression in order to transfer to different courses and qualifications must also be a priority.

7. The introduction of new courses and structures could provide additional opportunities for addressing the issues outlined above. However, it is important that the directions taken do not fall into the trap of providing materials, structures and resources which only tackle immediate day to day problems and, as a consequence, become redundant as contexts change. We need to make strides in finding cost effective measures which give insight and guidance in putting the notion of 'transferable numerical skills' into a practical proposition within both the academic and vocational courses on offer.

8. Increasing students' perception and competency with numerical and graphical skills is essential if we are to move towards a mathematically fluent society. This could be achieved by creating opportunities within courses, which would enable students to:

- be fluent with mental calculations and estimations;
- ascertain when an exact answer is needed and when an estimate is more appropriate;
- decide which mathematical operations are appropriate in particular contexts;
- use a calculator/computer correctly, confidently and appropriately;
- possess an ability to verify mental or calculator results;
- interpret and use tables, diagrams, graphs, statistical techniques and spreadsheets to organize, analyze and present information;
- interpret and validate quantitative results presented by others;
- formulate specific questions from vague problems;
- select effective problem solving strategies.

9. The implications for changes in teaching styles and materials in fostering the above skills are great. It would certainly call into question the conventional method of designing the curriculum in terms of topics followed by tests and examinations on these topics treated in an isolated and compartmentalized manner.

SYNTHESIS

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The purpose of sessions was to produce recommendations aimed at curriculum developers, textbook writers, editors, school superintendents, principals, teachers and researchers.

Two short presentations were made at the beginning of each of the first three sessions, followed by an hour discussion. This was possible because of the size of the group which averaged about 20 participants. The discussions moved from an initial emphasis on potential early school leavers and the ways to prevent them from leaving the formal educational system, to an emphasis on goals and strategies for providing additional educational opportunities for those who had left the formal system at some point. The last session was devoted to a discussion of principles, concepts, goals and strategies from the last point of view. The text which follows is a summary of these discussions.

Concepts

Numeracy: Skills of numerical and spatial thinking required for daily living in society.

Numeracy: Informed numeracy involves knowing when to use math, what math to use, how to do math, and how to use the results provided by the mathematics.

Numeracy: Deployment of simple mathematical processes and skills in the tackling and solving of real life problems and tasks. (Shell Center, Nottingham).

Numeracy: Math skills that enable to cope with the practical demands of everyday life (Cockroft Report).

Numeracy: Ability to reason intelligently with qualitative information (James Fey).

The reverse, innumeracy, has been defined by John Allen Paulos as the inability to understand and criticize numerical and statistical data, which leads people to fall in all sorts of scams and deceptions.

Principles

- Every student has the right to acquire (in or out of school) mathematical knowledge that is useful as universal knowledge; hence *every* student must have access to the full mathematics curriculum.
- All teachers have the duty to accept the former principle, in spite of the difficulty of identifying mathematical properties in the students' procedures, and of negotiating with them the acceptance of their knowledge as valuable.
- Objects and materials involve different levels of representation, and require knowledge of rules appropriate to their structure, which is often of symbolic nature.
- The construction of mathematical knowledge is a long and slow process, and it is an eminently social activity, mediated by language.
- Contextualization should take into account the use of standard and non-standard representations of the thinking process.
- Contextualization should take into account the individual's cultural and social environment rather than thwarting it.
- Types of mathematical thought citizen will need are now changing.
- Increasing demands of workplace for more math knowledge.
- Need for different ways to teach math because of the presence of calculators and computers.
- Affect and self esteem are very important.
- Contextualization should not mean leaving students immersed in their social/cultural situations and strategies, but lead the way to decontextualizing them, making them more abstract, general and complex, and thus more widely applicable, enabling students to mathematize in a more scientific way.
- A minimum core curriculum for all is desirable if it is rich enough to be compatible with the above principles, and if care is taken that the minimum does not become maximum for the majority of students.

Two questions

Why should a student learn math?

To the "Why" question we could point to the acquisition of career-related skills, of attitudes and knowledge which will result in an informed member of society, and will be part of an educated citizen.

What should they learn?

- Math as problem solving;
- Math as communication;
- Math as reasoning;
- Mathematical connections;
- Basic operations with whole numbers, fractions and decimals, statistics and probability, geometry and algebra.

Goals

General goals

- Empower to respond to demands of a society as it develops technologically;
- Ability to think, ability to ask questions and assess answers, cooperate, communicate, make decisions, improve self-discipline, imagination, resourcefulness, independence;
- Teach math skills, and skills to learn math;
- Achieve mathematical reasoning by formulating hypotheses, selecting proof elements, constructing arguments, extracting conclusions, mastering the mathematical pathway;
- Help students survive in life after school;
- Help keep students in school;
- Make students aware of resources and instruments available to them from their cultural and experiential background, making explicit their connection to mathematics, and also decontextualizing them;
- While at school, get students to see mathematics as a tool for solving many kinds of problems;

- Value math, communicate mathematically, reason mathematically to become confident in ability to do math, and math problem solvers;
- Acquire reading comprehension skills to understand texts with mathematical content across the curriculum and in daily life;
- Develop mathematical reasoning;
- Develop mathematical language;
- Develop problem solving strategies;
- Expand world view of the role of mathematics.

Specific goals

- Be fluent with mental calculations and estimations;
- Ascertain when an exact answer is needed and when an estimate is more appropriate;
- Decide which mathematical operations are appropriate in particular contexts;
- Use a calculator or computer correctly, confidently, and appropriately;
- Possess an ability to verify mental or calculator results;
- Interpret and use tables, diagrams, graphs, statistical techniques and spreadsheets to organize, analyze, and present information;
- Interpret and validate quantitative results presented by others;
- Formulate specific questions from vague problems;
- Select effective problem solving strategies.

Srategies

At policy making level

- Coordinate policies and activities with other agencies, especially employers, associations of principals, supervisors, teachers, organized minority groups, etc.

— Change assessment and prevent it from driving the curriculum to the teaching of routine calculation skills. The assessment should include practical projects, research projects, portfolios and open questions.

At teacher education level

— Improvement of math knowledge of teachers;
— Get teachers to accept that math knowledge of their students is also valid;
— Encourage teachers to appreciate the value of errors as possible generalizations out of limited experiences or as coming from different interpretations.

At classroom level

— Approach same content from all the mathematical aspects concerned with it;
— Approach same content at different levels of complexity and with different types of representation, including those used by mass media (tables, graphics, etc.);
— Consider school mistakes as raw material, useful in the process of knowledge construction;
— Let rigor be an effect of the activity, arising from the need to communicate, and not as a previous condition;
— Let evaluation be an integration of all the productive moments of a process, where the students evaluate themselves; the groups evaluate themselves and each student, and the teachers evaluate the adequacy of activities, topics, content, methods and evaluations;
— Pay more attention to everyday language;
— The emphasis in learning activities should be put on the operation carried out and not on the objects;
— Take into account not only social relations generated in society, but also those the school system causes;

- Projects with contextualized activities in such a way as to promote an instrumental conception of mathematics.[Example: Space experiment about tomato growth in space ship led students to compare speed in solving tangram puzzles in silence vs listening to Heavy Metal music: their hypothesis was that they could work better with silence];
- Require involvement of parents and extensive participation of teachers (Remediation is no remedy, in Educational Leadership, March 1989, John M. Peterson);
- Applications and connections with other areas of life and the school curriculum (implicit and explicit): Ask what they saw on TV, newspapers, and what they would like to understand better;
- Use of a cooperative learning environment;
- Learning activities characterized by high expectation and high status for participants;
- Calculators and computers as integral part of the mathematics classroom;
- Develop strategies that come from real contexts;
- Integrate everyday mathematical procedures with school mathematical knowledge;
- Operations on concrete cultural objects, like money, should be preferred; but if the students' daily life doesn't offer a sufficiently rich source of materials, teachers can take other cultural objects from their reserves;
- Use their own profession, trade, etc. to start the math curriculum. [Example: For adults in urban settings, use decoration, budget, rent, charge accounts, paychecks];
- Prepare for standardized tests only a short time before them, and realize that students do as well or better than when 'taught to the test' for a long period.

APPENDIX 1

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